Stepped Spillways: Technical Advance from 1900

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ABSTRACT: Stepped spillways are currently a standard design in hydraulic engineering. Whereas these spillways were hardly employed until the advent to the Roller-Compacted Concrete (RCC) Technology in the 1980s, they were of particular interest from the 1990s for spillways of small or medium unit discharge, provided the RCC machinery was available. This paper reviews the technical background of the hydraulic developments of the stepped spillway concept, from roughly 1900 until 1990, when large research projects set in due to the success of the stepped spillway technology. The main steps in the integration of stepped spillways in dam schemes are described, along with isolated research conducted with these and similar structures, so that this paper may be considered an aid in the understanding of the fundamental hydraulic background of stepped spillways. The biographies of the main actors in this research field are also highlighted, including a portrait of the persons concerned.

KEY WORDS: Biography, History, Hydraulics, RCC-Technology, Stepped Spillway.

1 INTRODUCTION

Stepped spillways are an old hydraulic element to convey floods over a small spillway. Copied from mountainous brooks, in which the step-pool geometry may be observed, combined with white water due to significant air entrainment, the concept was successfully applied in early dam engineering. As in the natural case, where flow velocities are known to be relatively small along a cascade of ‘steps’, the reduction of kinetic energy at the base of a spillways was found to be so small that the energy dissipator length could be reduced much below these of smooth spillways. The concept of stepped spillway was used throughout the 19th century, but the most prominent application certainly was that at the New Croton Dam (Fig. 1).

Whereas the Old Croton Aqueduct was erected from 1837 to 1842, to secure the water supply of New York City, the New Croton Aqueduct was built from 1885 to 1890, whose capacity was three times larger than that of the original design, thus satisfying the increasing demand of drinking water of the large city at the east coast. The New Croton Dam is located 5 km upstream from the mouth of the Croton River into Hudson River, and 5 km downstream of the Old Croton Dam, whose reservoir capacity was almost fifteen times smaller than that of the new Dam. The Dam should originally consist of a 185 m long straight wall, combined with a bent, 300 m long wall portion over which the spillway should be erected. The final design was slightly modified, with the spillway discharging into a side channel that collects and then discharges the flood water. The spillway is made as a stepped spillway counting to the largest built up to this time, with a maximum design discharge of 1,550 m^3/s. The design concept is described in detail by its designer, Edward Wegmann (1850-1935) in his book (Wegmann 1911).

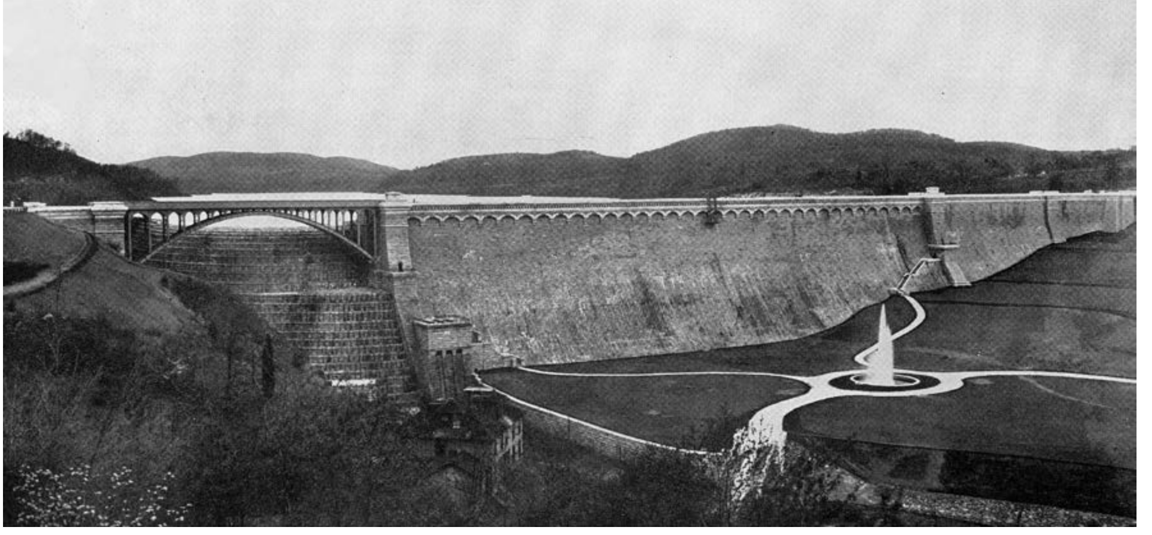
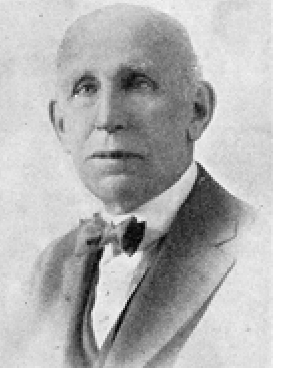


Figure 1 Tailwater view of New Croton Dam (Wegmann 1911)



Edward Wegmann was born on 27.11. 1850 in Rio de Janeiro of Swiss parents, and he passed away on 03.01. 1935 at Yonkers NY. He obtained the civil engineering degree in 1871 from the University of New York. After work for railroad companies he became in 1884 assistant engineer of the New Croton Aqueduct, designing first the Quaker Bridge Dam on Croton River, then with 90 m the highest dam of the world. His notable book The design and construction of dams, which appeared in 8 editions up to his death, was first published in 1888, constituting one of the earliest accounts on this novel technology. Wegmann was called in 1892 to the New Croton Dam Project, involving a number of dams and the existing Old Croton Dam. His monumental work Water supply of the city of New York, which describes the entire monumental project, was published in 1896. Wegmann was transferred in 1907 to the headquarters of the New Croton Company, New York, where he also wrote his final report on the work. He was appointed in 1910 chief engineer of the Aqueduct Commission, but in 1914 opened his own office in New York, where he remained until his death mainly as consultant of water supply projects. He also was Lecturer at Harvard University, Cambridge MA, and at Yale University, New Haven CT.

Stepped spillways were used throughout the 20th century, yet this was only applied in few cases, given that the smooth spillway was the standard design in this era, because of ease in construction, and increased spillway discharge capacity. This state was drastically changed with the introduction of the Roller-Compacted-Concrete (RCC) technology, which was originally applied in highway construction, but then soon adopted in hydraulic engineering. Concrete layers of typically 1 or 2 ft. were placed within short time, so that the dam structure consisting of a number of layers was simply, hydraulically efficient, and economically placed. Today, cascade spillways have also become a standard design in hydraulic engineering, so that it was considered worthwhile to review the first hundred years of this era, from the initiation of stepped spillways until the introduction of the RCC technology, roughly spanning from 1890 to 1990. Since then, significant advances both in concrete technology and in the hydraulic understanding were made, so that the history of stepped spillways before the RCC technology appears to be relevant for the understanding of the current success.

2 EARLY DEVELOPMENTS

An early hydraulic study on cascades, the so-called ‘step spillway’, was conducted analytically by Armani (1894). The flow was considered as a succession of either free or submerged spillways, thereby applying the basic weir flow formula. One of the plots shows a wavy surface structure which is reported to have occurred during a thunderstorm in the Vistula River Basin in 1893. A similar laboratory study was conducted by Koch (1923), yet without answering the main questions on the detailed hydraulic features of these flows.

Stepped spillways as currently employed became a design variation as compared with the standard spillways from 1900. Typical examples include the Gmünd Dam in Germany, then the largest dam in Europe (Küppers 1905), or the Eschbach Dam, the cascade of the Los Angeles Aqueduct involving the currently used stepped spillway (Heinly 1913), or the cascade spillways of the New Croton Dam of the New York City water supply (Hager 2008).

The first thorough study of stepped spillways was conducted by Gausmann and Madden (1923) on the Gilboa Dam, Gilboa NY, by the Board of Water Supply of New York City for the Catskill System. The dam is constituted by a 400 m long masonry section and a 300 m long earth dike. Its downstream side is formed of 6 m high steps; the 50 m high masonry portion was then the highest spillway of the world. Experiments were conducted on scales 1:50, 1:20, and 1:8 under a prototype approach flow head of 0.90 m (Fig. 2). Given the relatively small heads investigated, mainly so-called nappe flow was tested, for which the flow does not form a continuous configuration as for so-called skimming flow. The overfall section was – in contrast to current practice – shaped so that the jet was broken up, causing increased energy dissipation. It was found that all steps were impacted by the water flow for small discharges, whereas for larger discharges, the flow impacted only each second or even each third step, so that the step action was partially lost. Blocks were also positioned onto the steps, for which the general flow features were described as improved. The scatter between the findings resulting from the three model scales was partly large, particularly between the 1:20 and 1:50 model scales, which currently is attributed to scale effects. However, in the light of the knowledge on hydraulic models in the early 1920s, this study may be considered outstanding both in terms of experimentation, and the data analysis.

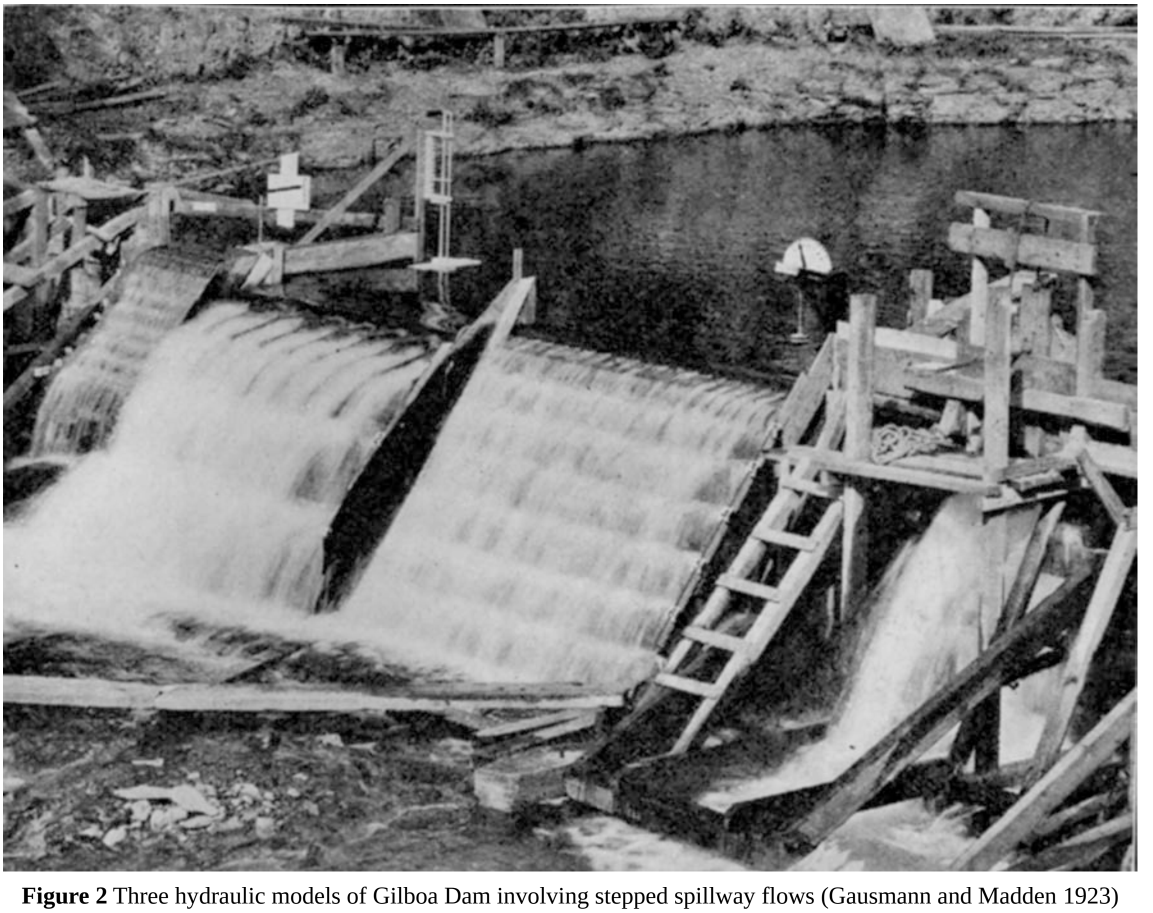


Figure 2 Three hydraulic models of Gilboa Dam involving stepped spillway flows (Gausmann and Madden 1923)

Lombardi and Marquenet (1950) conducted a study similar to that of Armani (1894), involving horizontal step portions much longer that the step height to control a torrent across a French village. The horizontal portions were made up of sediment, whose erosional pattern was determined using the

Meyer-Peter and Müller sediment transport formula. The solution proceeded by the use of a number of diagrams to obtain the main hydraulic parameters. The approach was verified with few experimental observations of a 1:30 scale model conducted at *Laboratoire National d'Hydraulique*, Chatou, France.

Roy Warner Gausmann was born on 01.09. 1882 at Brooklyn NY, and passed away on 01.06. 1974 at Newtown CT. He was educated as civil engineer at Columbia University, New York NY. From 1902 to 1925 he was in charge of the New York water supply up to division engineer. He also served on the construction of Ashokan and Gilboa Reservoirs, New York. He was general manager of all construction of the new water supply of the Greek capital Athens from 1925. From 1941 he served as assistant general manager of the US Overseas Division for the Ordnance Department in the Near East. He was further involved in hydraulic works at Aruba Island, at Lake Ontario, harbor studies in Liberia, and hydro-electric plants in Turkey, Ecuador, and Columbia. He was ASCE member, member of the US Military Engineers, and of the American Water Works Association AWWA.
The 1923 paper is concerned with hydraulic model experiments of the Gilboa Dam in Schoharie County NY. Of particular importance was the determination of the stepped spillway capacity. Special gauges were developed to measure both the lower and upper jet surfaces. A primitive scaling according to the current Froude number served as scale family indicator. This outstanding paper was awarded the 1923 ASCE James Laurie Prize.

Gaston Marquenet was born on 20.02. 1924 at Tours F, and passed away on 26.08. 1997 at Neuilly-sur-Seine F. He graduated as engineer from *Institut Electrotechnique* ENSEEHT, University of Toulouse, Toulouse F, in 1948, and there continued his education in mathematics, mechanics and fluid mechanics. He was from 1949 to 1954 a scientific collaborator of *Laboratoire National d'Hydraulique* and a member of the *Société Hydrotechnique de France* SHF. He was for the next six years a chief engineer at the *Centre d'Etudes et d'Organisation* CEO, organizing large engineering works. He was independent consultant within the Chamber of *Ingénieurs-Conseils de France* from 1960.
Marquenet's professional career was twofold, of which only the first portion is of interest here. His 1950 paper accounts for novel methods of flood protection in urban regions using cascade channels. Another paper was directed to a hydropower station on Vienne River, in which a channel junction was hydraulically investigated. The main hydraulic work conducted by Marquenet relates to air entrainment in shaft spillways, as published in the Proceedings of the 5th IAHR Congress held in1953 at Minneapolis MN.

Dean F. Peterson, Jr., was born on 03.06. 1913 at Delta UT, and passed away on 21.03. 1989 at Logan UT. He was educated at State University of Utah, Salt Lake City UT, from where he obtained the BS degree in 1934, and the DSc hon. in 1978. The MS degree in civil engineering was obtained from Rensselaer Polytechnic, Troy NY, and the Dr. title in 1939. Dean was a junior hydraulic engineer with the US Geological Survey from 1937 to 1939, project engineer for the Upper Potomac River until 1941, then progress engineer until being appointed in 1946 associate professor of civil engineering at Utah State University. He was there from 1957 until retirement in 1976 professor, dean of engineering, and vice-president of research.
Peterson was teacher, researcher and consultant. He was for instance a consultant of water resources, Office of Science and Technology, presiding it in the mid-1960s, or chairman of the Utah Advisory Council on Science and Technology from 1973. He was Fellow of the American Geophysical Union AGU, and member of the American Society of Civil Engineers ASCE, acting from 1972 to 1974 as vice-president, and was winner of the 1968 Royce J. Tipton Award. His research interests were in irrigation and drainage mainly, and hydraulic engineering, with papers published in the ASCE and AGU journals.

Peterson and Mohanty (1960) proposed three flow types in steep open channels, namely tranquil, tumbling, and rapid flows, based on observations of both natural streams and laboratory flumes. The flume used was 0.60 m wide and 20 m long, set at bottom slopes up to 8.5%. Blocks or sills as roughness elements were placed into it with a spacing between 5 and 15 times the roughness element heights. The flow depth above the roughness elements correlated linearly with the critical flow depth for any block spacing. Given the relatively high bottom slope, roll waves were observed whenever the approach flow Froude number was in excess of 1.6. The flow patterns observed included either a supercritical flow in the element vicinity, with a hydraulic jump shortly upstream from the next element, or continuous subcritical flow, as described by a succession of submerged weirs. The purpose of this research was to study the effect of the roughness element characteristics on energy dissipation, which was only partially successful, given the limited results.

3 RESEARCH FROM 1960

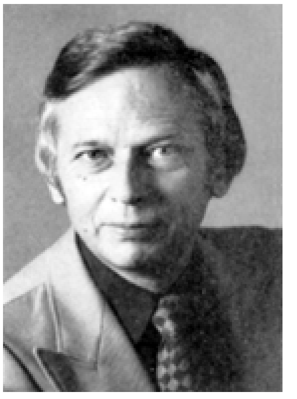
It appears that the first study of the current design of stepped spillways was conducted by Razvan (1966). One step arrangement involved a smooth crest portion, below which steps of constant step heights were added. It was found that the ratio between the pseudo-uniform air-water mixture flow depth and the computed uniform flow depth decreases as the discharge increases. Given the small step heights the flow was only partially aerated. Further data include the discharge coefficient, the discharge characteristics at the bottom of the stepped spillway, and the dissipation ratio of the stepped versus the smooth spillway arrangements. Aspects of cavitation damage are also addressed in this work and general design guidelines are provided. However, from a modern perspective, these are not amenable for a reliable design, given the exploratory character of the research.

Ernest Răzvan was born on 23.08. 1926 at Bucharest RO, and he passed away on 15.12. 1992 at Delft NL. After graduation as hydraulic engineer at Leningrad University, he joined the Bucharest Institute of Hydrotechnical Research. He was there promoted in the mid-1960s to Head of Department. In 1975, he immigrated to Israel joining Tahal, a major water works planning and design company. He thus became an expert of dam hydraulics and was abroad to supervise Tahal projects, as in Brazil, where he lectured also at the University of Sao Paolo. Răzvan spent his last years at the Delft Institute of Hydraulics. From the 1960s to 1975 he was Editor of *Institutul de Studii şi cercetări Hidrotehnice*.
Răzvan contributed to applied hydraulics mainly experimentally. From 1960 his researches were directed to turbulence effects of hydraulic jumps. These results were presented in IAHR Congresses, namely 1961 at Dubrovnik, 1967 at Fort Collins and 1971 at Paris. From the 1970s, Răzvan turned more to questions of fluvial hydraulics considering sediment entrainment and the effects of a dam on the upstream rivers. He also was active at the World Water Congresses.

Bretschneider and Geipert (1976) studied a novel type of cascade-shaped drops, which was stated to well-integrate into a landscape, but also perform hydraulically well. The entire drop structure is made up of a series of cup-shaped containers which are connected with an intermediate crest portion, so that the design has an almost sinusoidal longitudinal bottom profile with a certain elevation difference from the intake to the outlet. The design basis is stated to generate hydraulic jumps in the trough regions by which the energy increase due to the elevation difference is dissipated. The exact shape of the streamwise drop profile was optimized in laboratory tests; from a practical perspective, the result appears involved, so that cost limitations will be important, if considering that the average streamwise slope is lower than 10%, and that the bottom of the structure should have a fixed bed finish. No information on typical two-phase flow features is provided, including velocity and air concentration distributions.

Stephenson (1979a,b) proposed to stabilize channels and embankments with rockfill linings, given their resistance against high velocities and waves. He presented criteria for stone stability in flowing water under a comparatively large slope, along with the friction loss induced by the arrangement. The stability of lined rock elements was based on the equilibrium between the driving and retaining forces. The resulting equation for incipient stone motion is a generalized form of the Shields equation in river engineering, accounting for the increased bottom slope and the porosity. Next, the total head loss for a series of steps was determined, depending on the slope of the pseudo-bottom. Based on Essery and Horner (1978), a distinction between the nappe flow and skimming flow regimes was accounted for. Finally the paper includes a computational example in which the main steps are highlighted. It should be noted that the CIRIA Report appears to introduce the two notions ‘skimming flow’ and ‘stepped spillway’, both of which are currently a standard in stepped spillway flow studies.

Hans Bretschneider was born on 13.03. 1926 in Berlin, Germany. Hi joined in the 1950s the Institute of Water Resources, Technical University, Berlin, submitting in 1961 there his PhD thesis on broad-crested weir flow. He became in 1968 the successor of his former tutor, Heinrich Press (1901-1968), and there remained head of the Institute and professor of hydraulic engineering until his retirement in the late 1990s. He was awarded the honorary doctor degree in 1976, and became honorary president of the German Committee of Dams.
The main hydraulic interests of Bretschneider included the effect of surface forces on hydraulic laboratory modeling, similarity considerations of the Meyer-Peter and Müller sediment transport formula, crest geometries of frontal and lateral spillways, circular-shaped drop shafts including the Morning Glory Spillway, and features of air entrainment at bottom aerators. He was all through his career also interested in the German DIN standards, contributing mainly to the lingual definition of technical terms. He was from 1962 to 1977 editor of the German journal *Wasserwirtschaft*. Bretschneider currently lives in Berlin, Germany.



David Stephenson was born on 01.03. 1943 at South Shields UK, and he currently lives at Gaborone. He graduated as civil engineer from University of Witwatersrand, Johannesburg SA, in 1965, and there took the PhD degree in 1969. From 1966 he was assistant engineer of the Rand Water Board, Johannesburg SA. From 1972 to 1974 he was practicing engineer, accepting in 1977 at his Alma Mater the position of hydraulic engineering professor, acting from 1978 to 1980 as assistant director of its Hydrological Research Unit and from 1982 as director of the Water Systems Research Programme. From 2004 he was professor of water engineering, University of Botswana, Gaborone. Stephenson is member of the South African Institution of Civil Engineers, the American Society of Civil Engineers ASCE, and the Institution of Civil Engineers ICE, London UK. He is member of the International Association of Hydraulic Research IAHR, presiding in the 1990s its African Division.
Stephenson's main topics of research include hydrological modeling and pipe flow hydraulics. He also made research on water and sanitation problems of developing countries, flood studies in Africa, and flow modeling in pipelines.

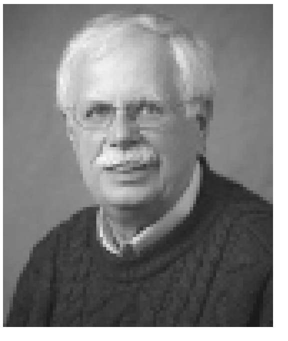
Young (1982) conducted a feasibility study of the stepped spillway project, the Upper Stillwater Dam, Utah, the first Roller-Compacted-Concrete RCC dam of the US Bureau of Reclamation USBR. It was found that the energy dissipation at the toe of the spillway was greatly reduced due to the step action, with maximum velocities of only 11 m/s at the 61 m high dam, so that only an 8 m long stilling basin was provided. The overall bottom slope of the stepped spillway is 0.6:1, at whose top is an un-gated 183 m long spillway, of 425 m^3/s design discharge. A hydraulic model study was conducted to investigate whether the proposed design was feasible, and to what degree the stepped spillway affected energy dissipation. Both a crest model and a toe model were erected at 1:5 scales. The final crest shape adopted involves steps of increasing height up to the point of tangency, so that the flow does not detach from the pseudo-bottom. Next, the flow on the stepped spillway was studied. It was found that a considerable spray develops along the steep flow portion, so that the side walls must be sufficiently high to contain the air-water mixture flow. Both the nappe and the skimming flow regimes were observed, depending on the specific discharges. Compared to two decades later, the employed instrumentation was poor, consisting of film cameras and Pitot tubes for velocity measurement. It was found that the energy loss at the base of the spillway amounted to only some 25% of the approach flow head, so that the step action was considered significant. The stilling basin adopted had less than half the length which would have been required for flow on the corresponding smooth spillway. Young therefore must be credited for having successfully introduced, with USBR, the stepped spillway technique including the RCC technique and the optimum crest to spillway transition.

It should be noted here that RCC is a special blend of concrete with essentially the same ingredients as conventional concrete but in different ratios. RCC is thus a mix of cement, water, sand, aggregate and common additives, but it contains much less cement and thus water, to allow for adequate placement using modified asphalt pavers, and then is compacted by vibratory rollers.

Marlene F. Young was born in 1955 in California. She graduated with the BSc degree in engineering in 1978 from the California State University, Long Beach CA, and then joined the US Bureau of Reclamation USBR, Denver CO, as project engineer. It was during this time where she was involved, as the first person, in the experimental investigation of the then novel Roller-Compacted-Concrete RCC dam. Her data formed the basis for the study of Sorensen (1985) described below. After having left USBR in 1982, she first was Public Works Director of Santa Barbara County, providing road, solid waste, building, and other county services. From 1992 to 2002 she then acted as Community Services Director at Lompoc CA, and from then to 2006 was City Manager of Solvang CA. After further positions as county engineer and project engineer, she is involved as civil engineer with governmental management in planning, public works, and other general administration at various locations in California. She currently lives in Santa Rosa CA.

Sorensen (1985) evaluated the performance of stepped spillway flow using a hydraulic model study. The spillway investigated was of Ogee crest shape, followed by steps of equal height except for the first few steps, to allow for a continuous transition from the crest to the constantly-sloping stepped spillway portion. The primary purposes of the study were to evaluate the effectiveness of the flow transition from the smooth to the stepped spillway portions, to quantify the energy dissipation, and to define the flow characteristics. It was found that stepped spillways are effective at dissipating considerable hydraulic energy, and that the transition proposed is simple to adjust in practice. The study related to the new Monksville Dam of the North Jersey District Water Supply Commission. It was recommended to further investigate an optimized step geometry, the face slope, and the crest elevation.

Robert M. Sorensen was born in 1938. He earned the BS degree from Newark College, Newark NJ, in 1960, the MS degree in 1962 from Lehigh University, Lehigh PA, in 1962, and the PhD degree from the University of California, Berkeley CA, in 1966. He obtained the PE degree in Texas in 1969. He was from the 1970s professor of hydraulic engineering at Lehigh University, and there became in 2009 professor emeritus of civil and environmental engineering.
The research interests of Sorensen include coastal engineering, tidal inlets, breakwaters, the effects of surface roughness on wave forces on piles, but also coastal sediment transport, fish passes and their environmental impact, scour around piles due to oscillatory wave motion, and of course stepped spillways. He took further interest on ship-generated waves, ship waves, and in 1993 presented a book entitled Basic wave mechanics. Given these topics, and his books, it may be stated that Sorensen rather by chance made his work on stepped spillways, because ‘his true love’ appears to be coastal hydraulics.



Rajaratnam (1990) analyzed the main features of skimming flow in stepped spillways using the data of Sorensen (1985). The coefficient of fluid friction c_f was found to range from 0.11 to 0.20, with an average of 0.18, whereas it increased to 0.27 for smaller discharges. This effect was explained with the two different regimes, namely the first under skimming flow, whereas the second applies for nappe flow. Using $c_f = 0.18$ allowed for determining the velocity at the toe of the spillway. The additional energy head loss due to the presence of the steps essentially depends on the square of the Froude number at the base of the spillway. For large Froude numbers, the ratio between the additional energy loss and the total energy head becomes simply 8/9, indicating an enormous contribution of the steps. This is also reflected by the extremely short stilling basin required for stepped spillways, as previously outlined. It was further found from the data of Essery and Horner (1978) that the onset of skimming flow occurs approximately if the critical flow depth $h_c = [q^2/(gb^2)]^{1/3}$ is larger than $0.8s$, with s as the step height, q as unit discharge, g as gravity acceleration, and b as spillway width. Similar findings were also provided by Stephenson (1990). These results have of course become subject of (slight) modification in the past two decades, given the enormous development in instrumentation, and the increased research activity in this field of hydraulic engineering.

4 CONCLUSIONS

Stepped spillways are currently a standard design in hydraulic engineering, mainly based on the successful development of the Roller-Compacted-Concrete RCC Technology. Copying nature from steep mountainous rivers, the principle of cascades was integrated at few dams already in the 19th century, without any hydraulic knowledge for a long time. From the 1920s, the first models of stepped spillways were investigated, but it was only in the 1980s when the first current designs were studied in laboratory models. It was also during this decade when notions as pseudo-bottom, skimming flow, or stepped spillway were accepted in the engineering community. This research is concerned with the hydraulic advances of stepped spillways until 1990, when the large research campaigns were initiated to answer the main questions relating to the optimum transition from the crest to the stepped portions, the step height and its effect on the flow regime, the aeration characteristics and the velocity and air concentration profiles, the uniform-aerated flow zone and its inset, and the tailwater characteristics including the aspects of energy dissipation. These advances made from 1990 need to be summarized in additional work.

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